W. Najm W.N.
J. Kasamani

AMERICAN UNIVERSITY OF BEIRUT
FACULTY OF ENGINEERING AND ARCHITECTURE
MECH 230 - DYNAMICS – FINAL EXAM. – FALL 2011

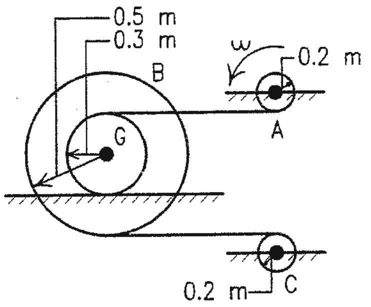
NAME: **ID No:** **Jan. 30, 2012**

2.5 hours open book Exam.

- Solve in sequence on the answer booklet.
- The question sheet in not considered in grading.
- Write clearly. Clarity is important in grading.
- Vectors are indicated in bold.
- Take $g=9.81 \text{ m/s}^2$ or $g=32.2 \text{ ft/s}^2$.

No.1– (20%)

The cable from drum A turns the double wheel B, which rolls on its hubs without slipping. At the shown instant, the angular velocity and the angular acceleration of A are respectively 4 rad/s and 3 rad/s^2 respectively, both in the counterclockwise direction. Note that the centers of pulleys A and C are fixed.



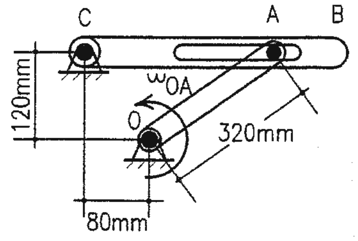
5% 1- Determine the angular velocity of the double wheel B at the instant shown.

5% 2- Determine the angular velocity of the drum C at the instant shown.

10% 3- Determine the angular acceleration of the double wheel B at the instant shown.

No.2– (20%)

Link OA has an angular velocity of 8 rad/s as it passes the position shown.

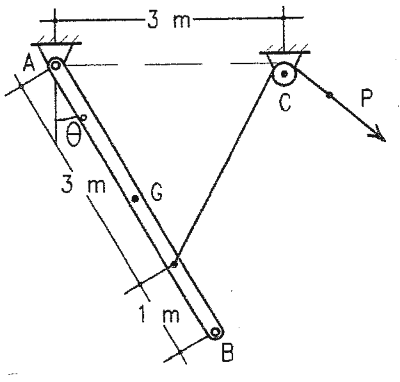


10% 1- Determine the angular velocity of link CB.

10% 2- Determine the Corriolis component of acceleration of pin A.

No.3– (20%)

The uniform 100 kg beam is freely hinged about its upper end A and is initially at rest in the vertical position with θ=0°. A force of 300 N is applied on the attached cable as shown.

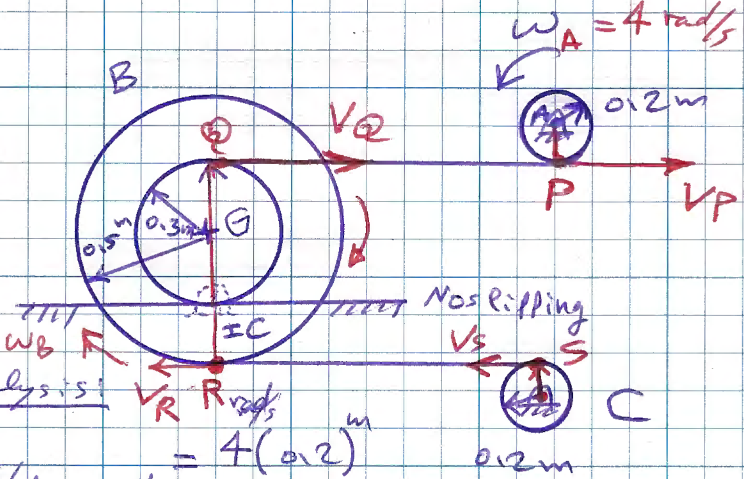


5% 1- Draw the free body diagram and the kinetic diagram at the instant θ=0°.

problem 1 : No slipping

Given: $\omega_A = 4$ rad/s ↺ Find ω_B , ω_C , α_B , α_C

$\alpha_A = 3$ rad/s² ↺



Velocity Analysis:

1\. $V_P = \omega_A r_{P/A\,\text{Fixed}} = 4(0.2)^m = 0.8$ m/s

$V_P = V_Q = 0.8$ m/s ; $V_Q = \omega_B r_{Q/IC}$ [on doublewheel B]

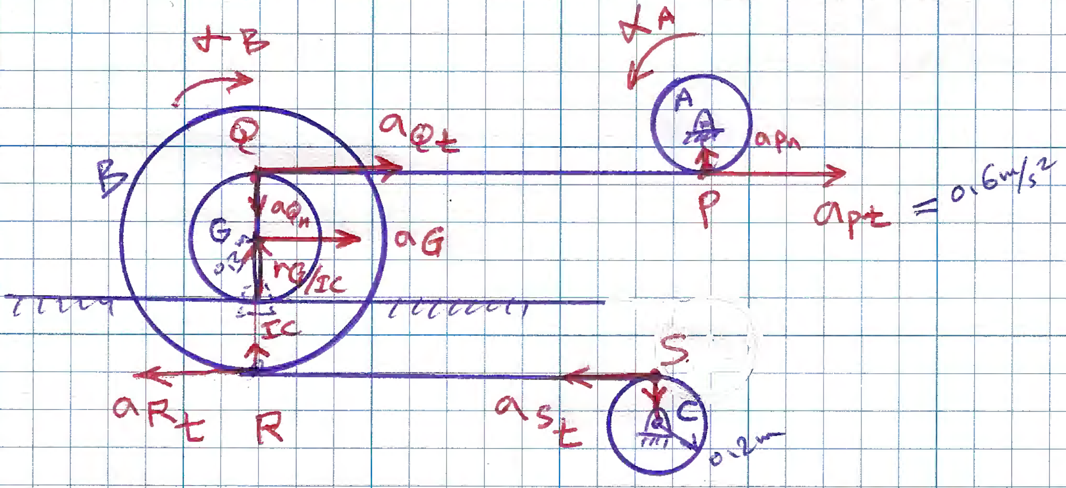
$\omega_B = \frac{V_Q}{r_{Q/IC}} = \frac{0.8\ m/s}{0.6\ m} = 1.3\bar{3}$ rad/s ↻ Ans.

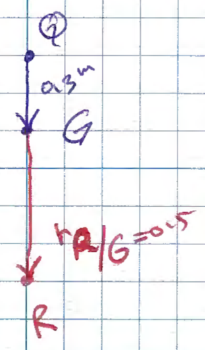
2\. $V_R = \omega_B r_{R/IC} = 1.3\bar{3}^{\,rad/s}(0.2^m) = 0.2667$ m/s

$V_S = V_R = 0.2667$ m/s ; $V_S = \omega_C r_{S/C}$; $\omega_C = \frac{V_S}{r_{S/C}} = \frac{0.2667}{0.2}$

$\omega_C = 1.3\bar{3}$ rad/s ↺ Ans.

3\.



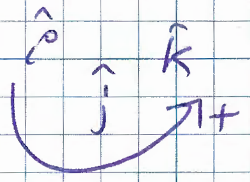


3. Pulley A: $a_{P_t} = \alpha_A r_{P/A} = \overset{rad/s^2}{3}\,(0.2)^{m} = 0.6\ m/s^2 \rightarrow$

$a_{P_t} = a_{Q_t} = 0.6\ m/s^2 \rightarrow$; $a_G = \alpha_B r_{G/IC} = 0.3\alpha_B$

$a_G = 0.3\alpha_B$

Consider Double wheel B:



$$\vec{a}_G = \vec{a}_Q + \vec{\alpha}_B \times \vec{r}_{G/Q} - \omega_B^2\, \vec{r}_{G/Q}$$

$$\underbrace{0.3\alpha_B}\,\hat{i} = 0.6\hat{i} - a_{Q_n}\hat{j} - \alpha_B\hat{k} \times (-0.3\hat{j}) - (1.3\bar{3})^2(-0.3\hat{j})$$

$$a_G\hat{i} = 0.6\hat{i} - a_{Q_n}\hat{j} - 0.3\alpha_B\hat{i} + 0.53\bar{3}\,\hat{j}$$

$\hat{i}$: $a_G = 0.6 - 0.3\alpha_B$

$0.3\alpha_B = 0.6 - 0.3\alpha_B$; $0.3\alpha_B + 0.3\alpha_B = 0.6$

$0.6\alpha_B = 0.6$; $\alpha_B = 1\ rad/s^2$ ↻ Ans. ; $a_G = 0.3(1)$

$a_G = 0.3\ m/s^2 \rightarrow$

4. $$\vec{a}_R = \vec{a}_G + \vec{\alpha}_B \times \vec{r}_{R/G} - \omega_B^2\, \vec{r}_{R/G}$$

$$-a_{R_t}\hat{i} + a_{R_n}\hat{j} = 0.3\hat{i} - \hat{k} \times (-0.5\hat{j}) - (1.33)^2(-0.5\hat{j})$$

$$= 0.3\hat{i} - 0.5\hat{i} + 0.88445\,\hat{j}$$

$\hat{i}$: $-a_{R_t} = 0.3 - 0.5$; $a_{R_t} = 0.2\ m/s^2 \leftarrow$

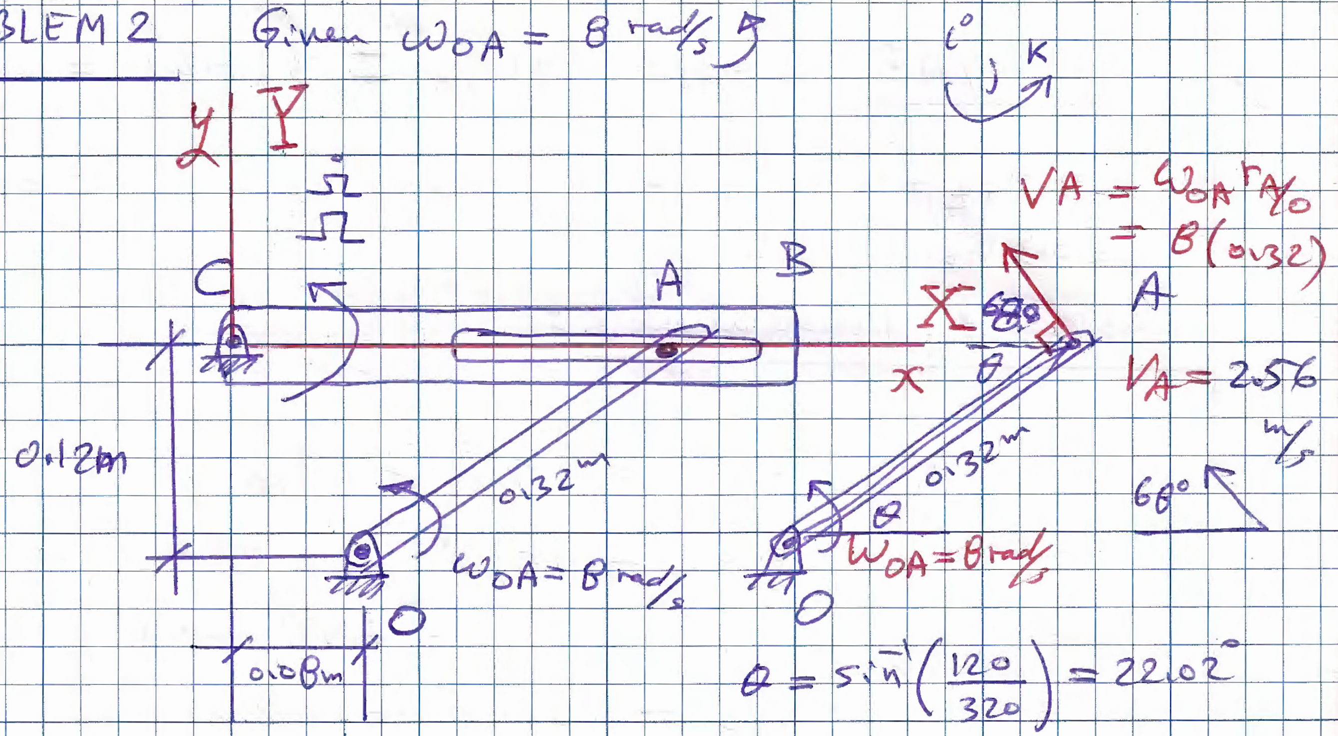
$a_{s_t} = a_{R_t} = 0.2\ m/s^2$

Consider pulley C: $a_{s_t} = \alpha_C r_{s/C}$; $\alpha_C = \dfrac{0.2\ m/s^2}{0.2\ m} = 1\ rad/s^2$

$\alpha_C = 1\ rad/s^2$ ↺ Ans.

PROBLEM 2

Given $\omega_{OA} = 8 \text{ rad/s} \circlearrowright$

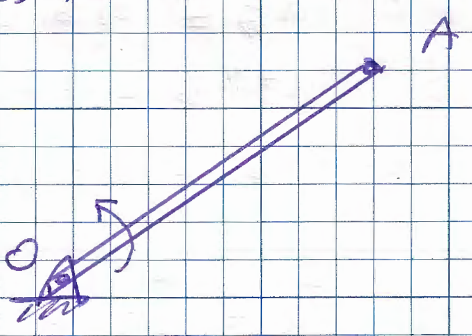


$V_A = \omega_{OA} r_{A/O}$
$= 8(0.32)$

$V_A = 2.56$ m/s

$\theta = \sin^{-1}\left(\frac{120}{320}\right) = 22.02^\circ$

$CA = 80^{mm} + 320 \cos 22.02^\circ = 376.657$ mm



Motion of rotating axes:

$\vec{V}_C = \vec{0}$ Fixed

$\vec{a}_C = \vec{0}$ Fixed

$\vec{\Omega} = \omega_{CB} \hat{K}$

$\dot{\vec{\Omega}} =$

Motion of A w.r.t rotating axes

$\vec{r}_{A/C} = 0.376657 \hat{i}$ m

$(\vec{V}_{A/C})_{xyz} = (V_{A/C})_{xy} \hat{i}$

$(\vec{a}_{A/C})_{xyz}$

$$\vec{V}_A = \vec{V}_C + \vec{\Omega} \times \vec{r}_{A/C} + (\vec{V}_{A/C})_{xyz}$$

$$-2.56 \cos 68^\circ \hat{i} + 2.56 \sin 68^\circ \hat{j} = 0 + \omega_{CB} \hat{K} \times 0.376657 \hat{i}$$

$$= 0.376657\, \omega_{CB} \hat{j} + (V_{A/C})_{xyz} \hat{i} \quad \boxed{+ (V_{A/C})_{xyz} \hat{i}}$$

$\hat{j}$: $2.56 \sin 68^\circ = 0.376657\, \omega_{CB}$

$\omega_{CB} = 6.3017 \text{ rad/s} \circlearrowleft$ Ans.

$\hat{i}$: $(V_{A/C})_{xyz} = -0.959$ m/s

; $2\vec{\Omega} \times \vec{V}_{A/C}$
$= 2(6.3017 \hat{k}) \times -0.959 \hat{i} = \underline{\underline{-12.087 \hat{j}}}$ m/s²

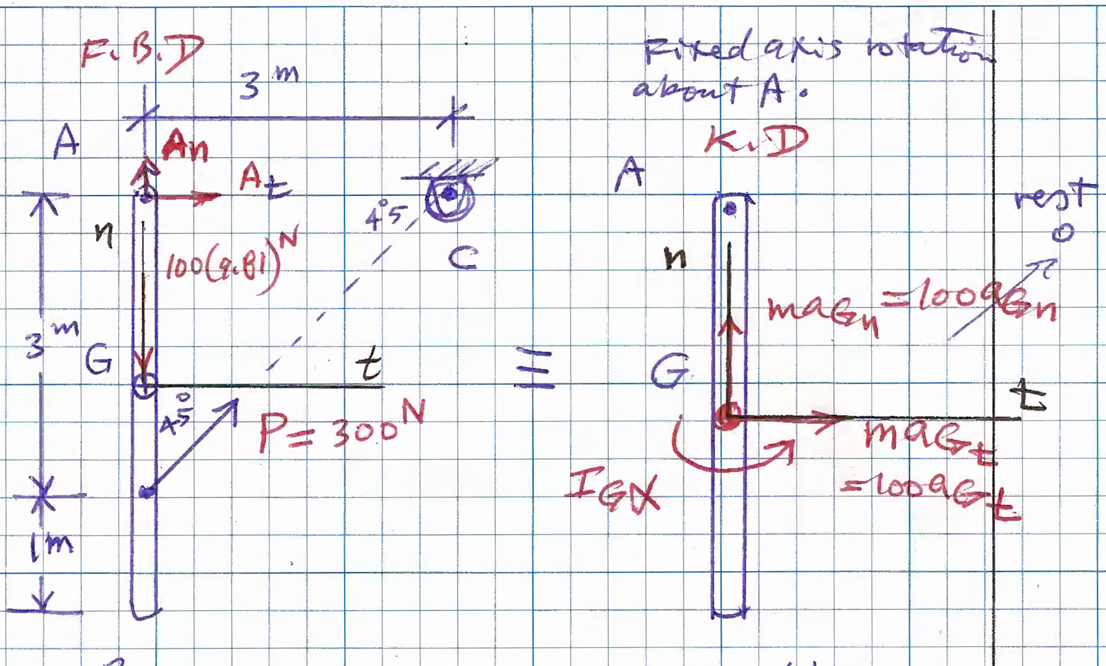
Problem 3:

Given $m = 100$ kg

@ $\theta = 0°$

1. Draw F.B.D & K.D

Fixed axis rotation about A.



$a_{G_t} = \alpha r_{G/A}$

$a_{G_t} = 2\alpha_{AB}$
$= 2(1.1932)$

2. $\circlearrowleft + \Sigma M_A = I_A\alpha$; A is a Fixed axis of rotation

$300^N \sin 45° (3^m) = \left[\frac{1}{3}(100)4^2\right]\alpha_{AB}$

$636.396 = 533.\bar{3}\,\alpha_{AB}$

$\alpha_{AB} = 1.1932$ rad/s² ↺ Ans.

$I_A = \frac{1}{3}ml^2$
$= \frac{1}{3}(100)4^2$
$= 533.\bar{3}$ kg m²

3. $+\uparrow \Sigma F_n = ma_{G_n}$; $300\cos 45° - 981^N + A_n = 0$; $A_n = 768.868^N \uparrow$

$\xrightarrow{+} \Sigma F_t = ma_{G_t}$; $300\sin 45° + A_t = 100(2)(1.1932)$

$A_t = 26.508$ N → ; $A = 769.325$ N ; $\alpha = 88°$ Ans.

4. Find ω_{AB} @ $\theta = 15°$

$T_1 + \Sigma U_{1-2} = T_2$; $T_1 = 0$ rest

$T_2 = \frac{1}{2}I_A\omega^2$; A is Fixed axis.

$= \frac{1}{2}\left[\frac{1}{3}(100)(4^2)\right]\omega_{AB}^2$

$T_2 = 266.67\,\omega_{AB}^2$

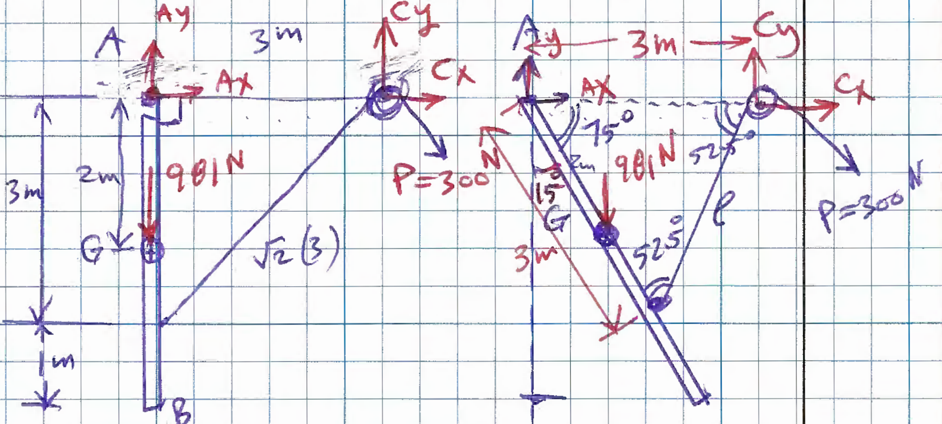
$\Sigma U_{1-2} = -981^N(\overset{\Delta y}{2 - 2\cos 15°})^m$

$+ 300^N(\sqrt{2}(3) - 3.6526)$

$= -66.8535 + 177.012 = 110.159$ J

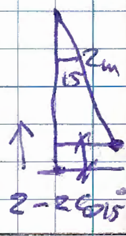
Substitute: $0 + 110.159^J = 266.67\,\omega_{AB}^2$

$\omega_{AB} = 0.6427$ rad/s ↺ Ans.



$\frac{l}{\sin 75°} = \frac{3}{\sin 52.5°}$

$l = 3.6526$ m



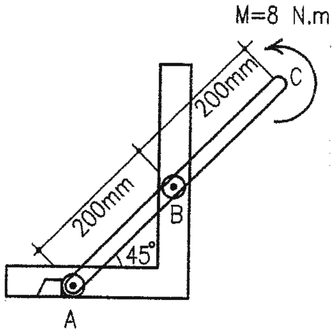
5% 2- Determine the initial angular acceleration of the beam at the instant θ=0°.

5% 3- Determine the horizontal and vertical components of the reactions at pin A at the instant θ=0°.

5% 4- Determine the angular velocity of the beam when θ=15°.

No.4– (20%)

The uniform 3 kg bar ABC is initially at rest with end A bearing against the stop in the horizontal guide. When a constant couple M=8 N.m is applied to end C, the bar rotates causing the end A to strike the side of the vertical guide with a velocity of 3 m/s. There is energy loss due to friction in the guide and the roller.

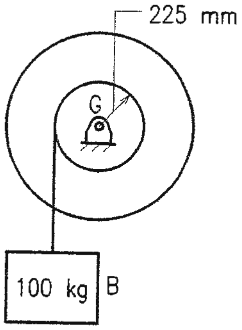


10% 1- Determine the angular velocity of the bar when it is in the vertical position .

10% 2- Determine the energy loss due to friction.

No.5– (20%)

The 320 kg flywheel of a small hoisting engine has a centroidal radius of gyration of 600 mm. The power is cut off when the angular velocity of the flywheel is 100 rpm clockwise. Block B has mass of 100 kg.



10% 1- Draw the impulse and momentum diagrams of the system.

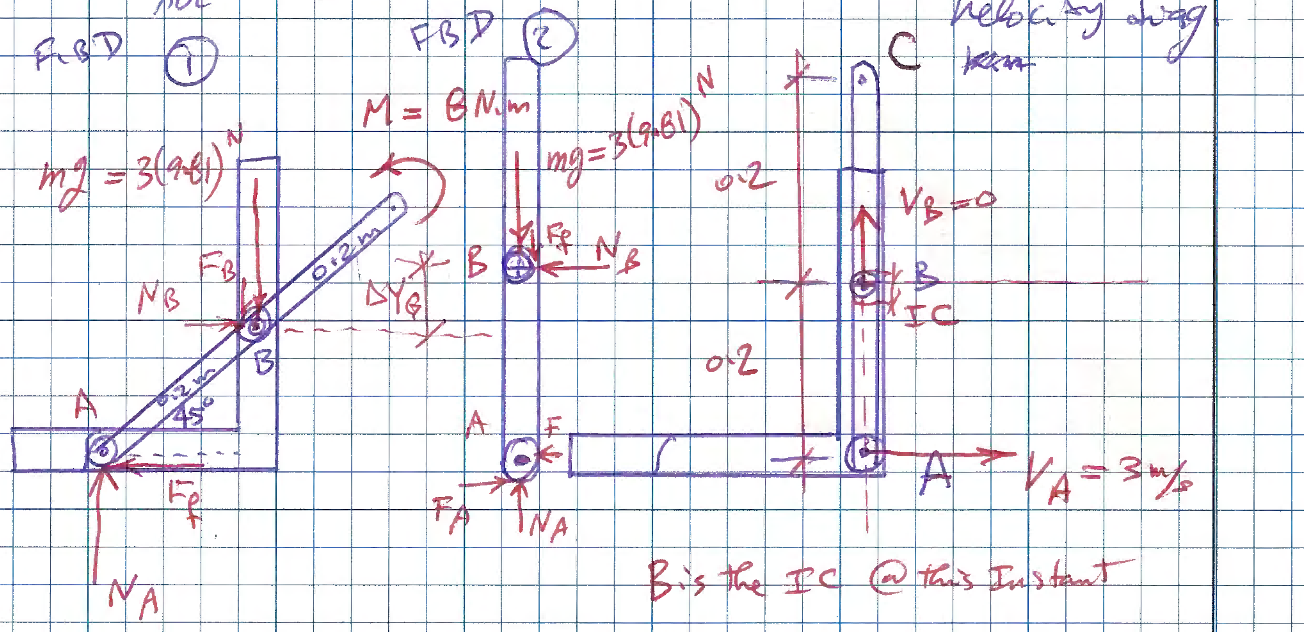
5% 2- Determine the time required for the system to come to rest.

5% 3- Determine the tension in the cord C.

PROBLEM 4 :

Given $m_{ABC} = 3kg$ ① @ rest

Abs. velocity diag



B is the IC @ this Instant

1_

$$I_G = \frac{1}{12} m l^2 = \frac{1}{12}(3)(0.4)^2 = 0.04\ kg.m^2$$

$\Rightarrow V_A = \omega_{ABC}\, r_{A/IC}$

$$\omega_{ABC} = \frac{V_A}{r_{A/IC}} = \frac{3}{0.2} = 15\ rad/s$$ Ans.

2_ $T_1 + \Sigma U_{1-2} = T_2$

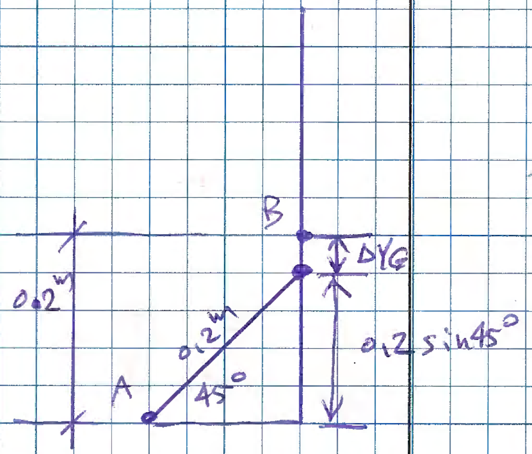
$T_1 = 0$ rest ; $T_2 = \frac{1}{2} I_{IC}\, \omega_{ABC}^2 = \frac{1}{2}\left[\frac{1}{12} m l^2\right] \omega_{ABC}^2$

$$T_2 = \frac{1}{2}\left[\frac{1}{12}\, 3^{kg}\ 0.4^2\right](15^2) = 4.5\ J$$

$$\Sigma U_{1-2} = mg\,\Delta Y_G + M\,\Delta\theta + U_{1-2\,friction}$$

$\Delta Y_G = 0.2 - 0.2\sin 45° = 0.2(1-\sin 45°)_m$ ↑

$\Delta\theta = \frac{\pi}{4}$



$$\therefore \Sigma U_{1-2} = -\overbrace{0.2(1-\sin 45°)\, 3\,(9.81)} + 8^{N.m}\left(\frac{\pi}{4}\right) + U_{1-2\,friction}$$

$$= -1.72397^J + 6.2832^J + U_{1-2\,friction}$$

$$= 4.5592 + U_{1-2\,friction}$$

substitute : $0 + 4.5592^J + U_{1-2\,friction} = 4.5\ J$

$$U_{1-2\,friction} = -0.05922\ J$$ Ans.

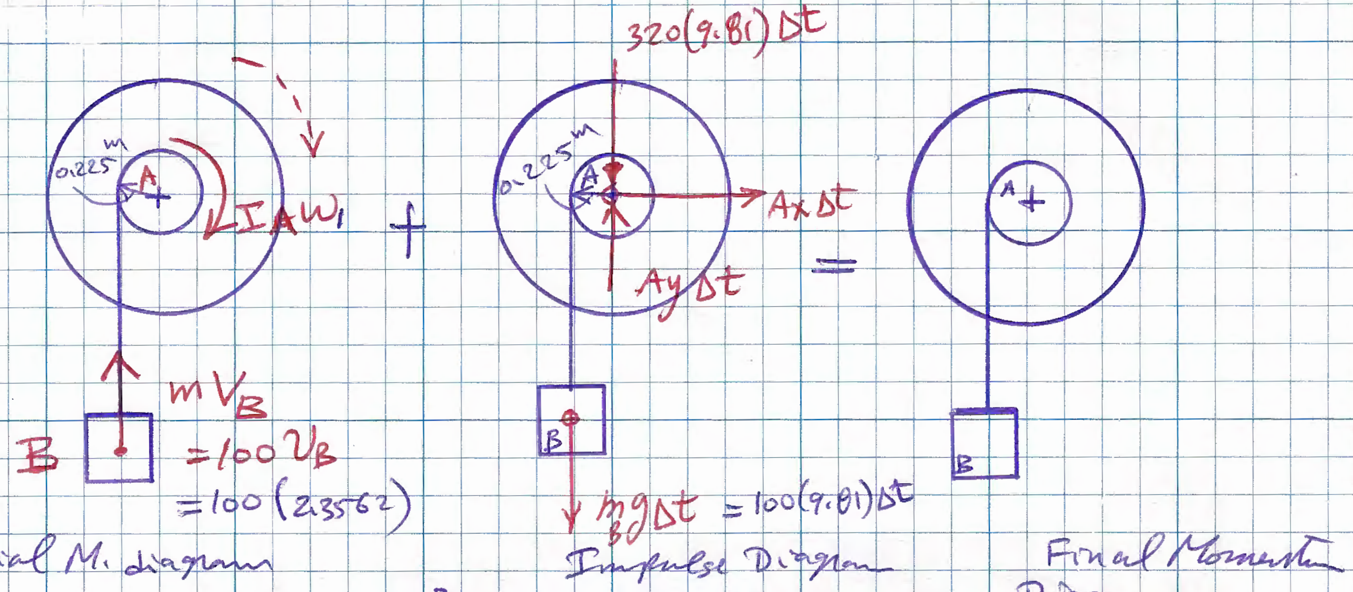
PROBLEMS : $m_A = 320\,kg$; $K_G = 600\,mm = 0.6\,m$

$$\omega_1 = \frac{100\ \cancel{rev}}{1\,min} \times \frac{2\pi\ rad/rev}{60\,s/min} = 10.472\ rad/s$$

$$V_B = \omega\, r_{B/A} = 10.472^{rad/s}\left(0.225^{m}\right)$$

$$V_B = 2.3562\ m/s \uparrow$$

1. Draw I.& M. diagrams:



$$I_A = m_A K_A^2 = 320(0.6)^2 = 115.2\ kg.m^2$$

2. Find Δt

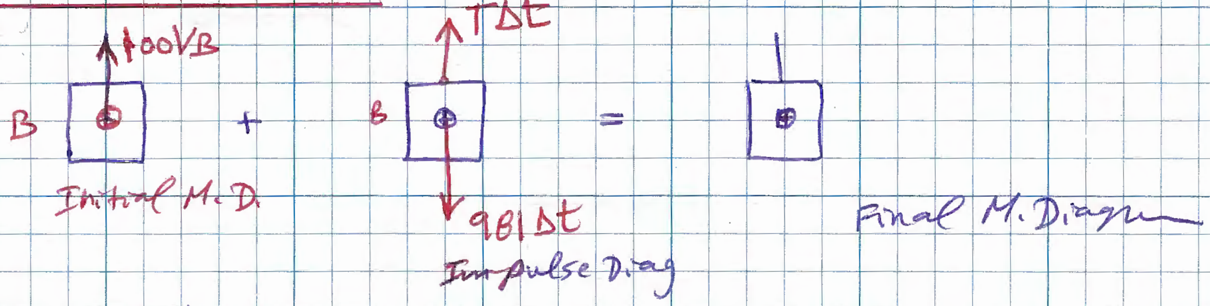
$\overset{\curvearrowright}{+}\ \Sigma(\text{moments})_A$:

$$\overset{kg.m^2/s}{I_A\omega_1} + \overset{kg.m/s}{100V_B}(0.225)^{m} \oplus - \overset{N.s}{100(9.81)\Delta t}\left(0.225^{m}\right) = 0$$

$$115.2\ kg.m^2 \times \left(10.472\ rad/s\right) + \overset{kg}{100}\left(2.3562^{m/s}\right)\left(0.225^{m}\right) = 981(0.225)\Delta t$$

$$\Delta t = 5.7057\ s \quad \text{Ans.}$$

3. Consider only Block B:



$+\uparrow\ \Sigma(\text{vectors})_y$:

$$\overset{kg}{100}\left(2.3562^{m/s}\right) + T(5.7057)^{s} - \overset{N}{981}\left(5.7057^{s}\right) = 0$$

$$T = 939.704\ N \quad \text{Ans.}$$